

p -BRANES, D -BRANES AND M -BRANES

E. BERGSHOEFF

*Institute for Theoretical Physics, University of Groningen,
Nijenborgh 4, 9747 AG Groningen, The Netherlands*

We consider solutions to the string effective action corresponding to p -Branes, D -Branes and M -Branes and discuss some of their properties.

1 Introduction

The original classification of super p -branes was based on the assumption that the embedding coordinates, in a physical gauge, form worldvolume scalar multiplets¹. A classification of such scalar multiplets with T (transverse) scalar degrees of freedom in $p + 1$ dimensions leads to the following table:

Table 1: Scalar multiplets with T scalars in $p + 1$ dimensions.

$p + 1$	T	T	T	T
1	1	2	4	8
2	1	2	4	8
3	1	2	4	8
4		2	4	
5			4	
6			4	

Each scalar multiplet corresponds to a p -brane in d target spacetime dimensions with

$$d = (p + 1) + T. \quad (1)$$

Here the target space has been divided into $p + 1$ worldvolume and T transverse directions. The Table describes 16 p -branes with $p \leq 5$ and $d \leq 11$.

The corresponding p -brane actions consist of a kinetic term and a Wess-Zumino (WZ) term. The kinetic term is given by

$$S_{\text{kin}}^{(p)} = \int d^{p+1}\xi \sqrt{|g|}, \quad (2)$$

where g is the determinant of the induced metric. The WZ term is given by the pull-back of a $(p + 1)$ -form potential. For the case of interest (10 dimensions)

these are a two-index Neveu-Schwarz/Neveu-Schwarz (NS/NS) tensor $B^{(1)}$ (heterotic, IIA or IIB one-brane) and the dual six-index tensor $\tilde{B}_{\text{het}}^{(1)}$ (heterotic five-brane):

$$S_{\text{WZ}}^{(1)} = \int d^2\xi B^{(1)}, \quad (3)$$

$$S_{\text{WZ}}^{(5)} = \int d^6\xi \tilde{B}_{\text{het}}^{(1)}, \quad (4)$$

where $d\tilde{B}_{\text{het}}^{(1)} = *dB^{(1)}$. Note that the action (3) is the same independent of whether the one-brane is propagating in a $N = 1$, IIA or IIB supergravity background.

2 Solutions

To each p -Brane given in Table 1 one can associate a solution corresponding to the following d -dimensional string-frame Lagrangian:

$$\mathcal{L}_{S,d} = \sqrt{|g|} \left\{ e^{-2\phi} [R - 4(\partial\phi)^2] + \frac{(-)^{p+1}}{2(p+2)!} e^{a\phi} F_{(p+2)}^2 \right\}. \quad (5)$$

Here ϕ is the dilaton and $F_{(p+2)}$ is the curvature of a $(p+1)$ -form gauge field

$$F_{(p+2)} = dA_{(p+1)}. \quad (6)$$

Note that we use a form of the action in which both the (electric) $(p+1)$ -form gauge field as well as the (magnetic) $(D-p-3)$ -form gauge field occurs. It is understood that the duality relation between them is only applied at the level of the equations of motion and not in the action. This is similar to the pseudo-action formulation of².

The general p -Brane solution is given by³

$$\begin{aligned} ds_{S,d}^2 &= H^\alpha dx_{(p+1)}^2 - H^\beta dx_{(D-p-1)}^2, \\ e^{2\phi} &= H^\gamma, \\ F_{0\dots pi} &= \delta \partial_i H^\epsilon, \end{aligned} \quad (7)$$

with the parameters α, \dots, ϵ given by

$$\alpha = \frac{1}{N}(2-a), \quad \beta = -\frac{1}{N}(2+a),$$

$$\begin{aligned}\gamma &= \frac{1}{N} \left[2(p+1) + (2+a)(1 - \frac{1}{2}d) \right], \\ \delta^2 &= -\frac{4}{N}, \quad \epsilon = -1,\end{aligned}\tag{8}$$

with $N = (p+1)a + (1 - \frac{1}{2}d)(1 + \frac{1}{2}a)^2$. In the case of p -branes it is assumed that the $(p+1)$ -form gauge field is a NS/NS field, i.e. $a = -2$ (+2) for the electric (magnetic) gauge-fields. For instance, the ten-dimensional one-brane, or fundamental string, solution is given by (take $d = 10, p = 1$ and $a = -2$)⁴

$$\begin{aligned}ds_{S,10}^2 &= H^{-1}dx_{(2)}^2 - dx_{(8)}^2, \\ e^{2\phi} &= H^{-1}, \\ F_{01i} &= \partial_i H^{-1},\end{aligned}\tag{9}$$

where $F_{(3)}$ is the curvature of the NS/NS tensor $B^{(1)}$. On the other hand, the ten-dimensional five-brane solution corresponds to (take $d = 10, p = 5$ and $a = +2$)

$$\begin{aligned}ds_{S,10}^2 &= dx_{(6)}^2 - Hdx_{(4)}^2, \\ e^{2\phi} &= H, \\ F_{012345i} &= \partial_i H^{-1}.\end{aligned}\tag{10}$$

Note that in the latter case we use the dual six-index tensor $\tilde{B}_{\text{het}}^{(1)}$.

Another example is provided by p -Brane solutions in eleven dimensions which we refer to as Mp -Branes⁵. Since there is no dilaton in this case, it is more convenient to work with the Einstein-frame Lagrangian

$$\mathcal{L}_{E,d} = \sqrt{|g|} \left[R + \frac{1}{2}(\partial\phi)^2 + \frac{(-)^{p+1}}{2(p+2)!} e^{a\phi} F_{(p+2)}^2 \right],\tag{11}$$

in which case the general p -Brane solution is given by

$$\begin{aligned}\alpha &= -\frac{4}{N}(d-p-3), \quad \beta = \frac{4}{N}(p+1), \\ \gamma &= \frac{4a}{N}(d-2), \quad \delta^2 = \frac{4}{N}(d-2), \quad \epsilon = -1,\end{aligned}\tag{12}$$

with $N = (d-2)a^2 + 2(p+1)(d-p-3)$. For instance, the $M2$ -Brane solution is given by (take $d = 11, p = 2$ and $a = 0$)⁶

$$\begin{aligned}
ds_{E,11}^2 &= H^{-2/3} dx_{(3)}^2 - H^{1/3} dx_{(8)}^2, \\
F_{012i} &= \partial_i H^{-1}.
\end{aligned} \tag{13}$$

Recently, two different extensions of the p -Branes classified in Table 1 have been given. The first extension concerns the general Mp -Branes in eleven dimensions. At the level of solutions, it is clear that there exists also a $M5$ -Brane solution given by (take $d=11$, $p=5$ and $a=0$)⁷

$$\begin{aligned}
ds_{E,11}^2 &= H^{-1/3} dx_{(6)}^2 - H^{2/3} dx_{(5)}^2, \\
F_{012345i} &= \partial_i H^{-1}.
\end{aligned} \tag{14}$$

The second extension concerns the so-called Dp -Branes⁸. At the level of solutions they correspond to p -Brane solutions whose charged is carried by a Ramond-Ramond (RR) gauge field, i.e. one which has $a=0$ in the string frame. The Dp -solutions are most easily formulated as solutions of the string-frame Lagrangian (5). These solutions are given by (take $d=10$, $a=0$)

$$\begin{aligned}
ds_{S,10}^2 &= H^{-1/2} dx_{(p+1)}^2 - H^{+1/2} dx_{(9-p)}^2, \\
e^{2\phi} &= H^{-\frac{1}{2}(p-3)}, \\
F_{0\dots pi} &= \partial_i H^{-1}.
\end{aligned} \tag{15}$$

3 Actions

The natural question arises why the $M5$ -brane and all the Dp -branes were absent in the original classification summarized in Table 1. As far as the Dp -Branes are concerned the answer is that these extended objects are described by embedding coordinates that, in a physical gauge, form worldvolume vector multiplets. A classification of all vector multiplets with T (transverse) scalars in $p+1$ dimensions is given by the table below.

In the case of ten dimensions, this leads to Dp -branes for $0 \leq p \leq 9$. The kinetic term of these Dp -branes is given by the following Born-Infeld type action:

$$S_{\text{kin}}^{(Dp)} = \int d^{p+1} \xi \, e^{-\phi} \sqrt{|\det(g_{ij} + \mathcal{F}_{ij})|}, \tag{16}$$

where g_{ij} is the embedding metric and $\mathcal{F} = 2dV - B^{(1)}$ is the curvature of the worldvolume gauge field V .

Table 2: Vector multiplets with T scalar degrees of freedom in $p + 1$ dimensions.

$p + 1$	T	T	T	T
1	2	3	5	9
2	1	2	4	8
3	0	1	3	7
4		0	2	6
5			1	5
6			0	4
7				3
8				2
9				1
10				0

There is also a WZ term which describes the coupling of the RR fields to the Dp -brane:

$$S_{\text{WZ}}^{(Dp)} = \int d^{p+1}\xi \mathcal{A} e^{\mathcal{F}}, \quad (17)$$

with $\mathcal{A} = \sum_{q=0}^9 A_{(q+1)}$. Here it is understood that, after expansion of the exponent, the $(p + 1)$ -form is picked out. In particular it means that the WZ-terms for $p \geq 3$ contain both electric as well as magnetic potentials⁹. To define the dual potential we need to specify whether the curved background is given by $N = 1$ or IIA/IIB supergravity. For instance, the dual potentials that couple to the heterotic 5-brane, the IIA five-brane (which follows from direct dimensional reduction of the eleven-dimensional five-brane) and the D5-brane are defined by, respectively,

$$\begin{aligned} *dB^{(1)} &= d\tilde{B}_{\text{het}}^{(1)}, \\ *dB^{(1)} &= d\tilde{B}_{\text{IIA}}^{(1)} - \frac{105}{4}CdC - 7A^{(1)}G(\tilde{C}), \\ *dB^{(1)} &= d\tilde{B}_{\text{IIB}}^{(1)} + DdB^{(2)} - \frac{1}{4}\epsilon^{kl}B^{(2)}B^{(k)}dB^{(l)}. \end{aligned} \quad (18)$$

Here $G(\tilde{C})$ is the curvature of the dual 5-form potential \tilde{C} defined in¹⁰.

Similarly, for the $M5$ -Brane the embedding coordinates, in a physical gauge, form a worldvolume tensor multiplet. This leads us to the table below. Note that the $M5$ -Brane in 7 dimensions has a one-dimensional transverse space and hence is not asymptotically flat. A non-trivial feature of these five-

Table 3: Tensor multiplets with T scalar degrees of freedom in $p + 1$ dimensions.

$p + 1$	T	T
6	1	5

branes is that the tensor multiplet contains a selfdual two-form. The kinetic term at quadratic order is given by ⁵

$$S_{\text{kin}}^{(M5)} = \int d^6\xi \sqrt{|g|} \left[1 + \frac{1}{2} \mathcal{H}^2 + O(\mathcal{H}^4) \right], \quad (19)$$

with $\mathcal{H} = 3(dW - 1/2C)$ and $\mathcal{H} = *\mathcal{H} + O(\mathcal{H}^3)$. On the other hand, the WZ term is given by ^{5,11,10}

$$S_{\text{WZ}}^{(M5)} = \int d^6\xi \left[\frac{1}{70} \tilde{C} + \frac{3}{4} \mathcal{H} C \right], \quad (20)$$

with the dual 6-form potential defined by ¹²

$$d\tilde{C} - \frac{105}{4} C dC = *dC. \quad (21)$$

It has been verified that, to quadratic order in \mathcal{H} , the $M5$ -brane action reduces to the $D4$ -brane action ¹⁰. The determination of the higher order in \mathcal{H} terms of the kinetic term remains an open issue (see, however, ¹³).

4 Supersymmetry

It is not difficult to verify that all p -, D - and M -brane solutions preserve half of the supersymmetry. Here we present a proof for the Dp -branes. In order to give a unified treatment it is convenient to treat the IIA and IIB supergravity theories at an equal footing (for more details, see ¹⁴). The relevant supersymmetry rules of the gravitino and dilatino ¹⁵ are given by (using the string-frame metric)

$$\begin{aligned} \delta\psi_\mu &= \partial_\mu \epsilon - \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon + \frac{(-)^p}{8(p+2)!} e^\phi F \cdot \gamma \gamma_\mu \epsilon'_{(p)}, \\ \delta\lambda &= \gamma^\mu (\partial_\mu \phi) \epsilon + \frac{3-p}{4(p+2)!} e^\phi F \cdot \gamma \epsilon'_{(p)}, \end{aligned} \quad (22)$$

with $F \cdot \gamma \equiv F_{\mu_1 \dots \mu_{p+2}} \gamma^{\mu_1 \dots \mu_{p+2}}$ and $\epsilon'_{(p)}$ given by

Table 4: Definition of the spinor $\epsilon'_{(p)}$.

p	$\epsilon'_{(p)}$ (IIA)	p	$\epsilon'_{(p)}$ (IIB)
0	ϵ	-1	$i\epsilon$
2	$\gamma_{11}\epsilon$	1	$i\epsilon^\star$
4	ϵ	3	$i\epsilon$
6	$\gamma_{11}\epsilon$	5	$i\epsilon^\star$
8	ϵ	7	$i\epsilon$

A straightforward calculation shows that the Killing spinor is given by

$$\epsilon = H^{-1/8}\epsilon_0, \quad \epsilon + \gamma_{01\dots p}\epsilon'_{(p)} = 0, \quad (23)$$

for constant spinor ϵ_0 .

5 Open Issues

Sofar we have encountered actions with worldvolume scalar, vector and tensor multiplets. Consider the string and five-brane actions in ten dimensions. For each p we expect a heterotic, IIA and two IIB actions. The table below shows that, restricting ourselves to worldvolume $(q+1)$ -form gauge fields with $q = -1, 0, 1^a$, we cannot describe the second IIB five-brane action. This action should, via $SL(2, R)$ duality, be related to the $D5$ -brane action. A preliminary study¹⁶ shows that the action involves a worldvolume 3-form gauge field, as indicated in the table. It would be of interest to construct this action.

Table 5: String and Five-brane actions with world-volume $(q+1)$ -form gauge fields.

q	$p = 1$	$p = 5$
-1	het, IIA, IIB	het
1	IIB	IIB
2,4		IIA
3		IIB ?

^aIn the case of the IIA five-brane there is a scalar that can be dualized to a 4-form.

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